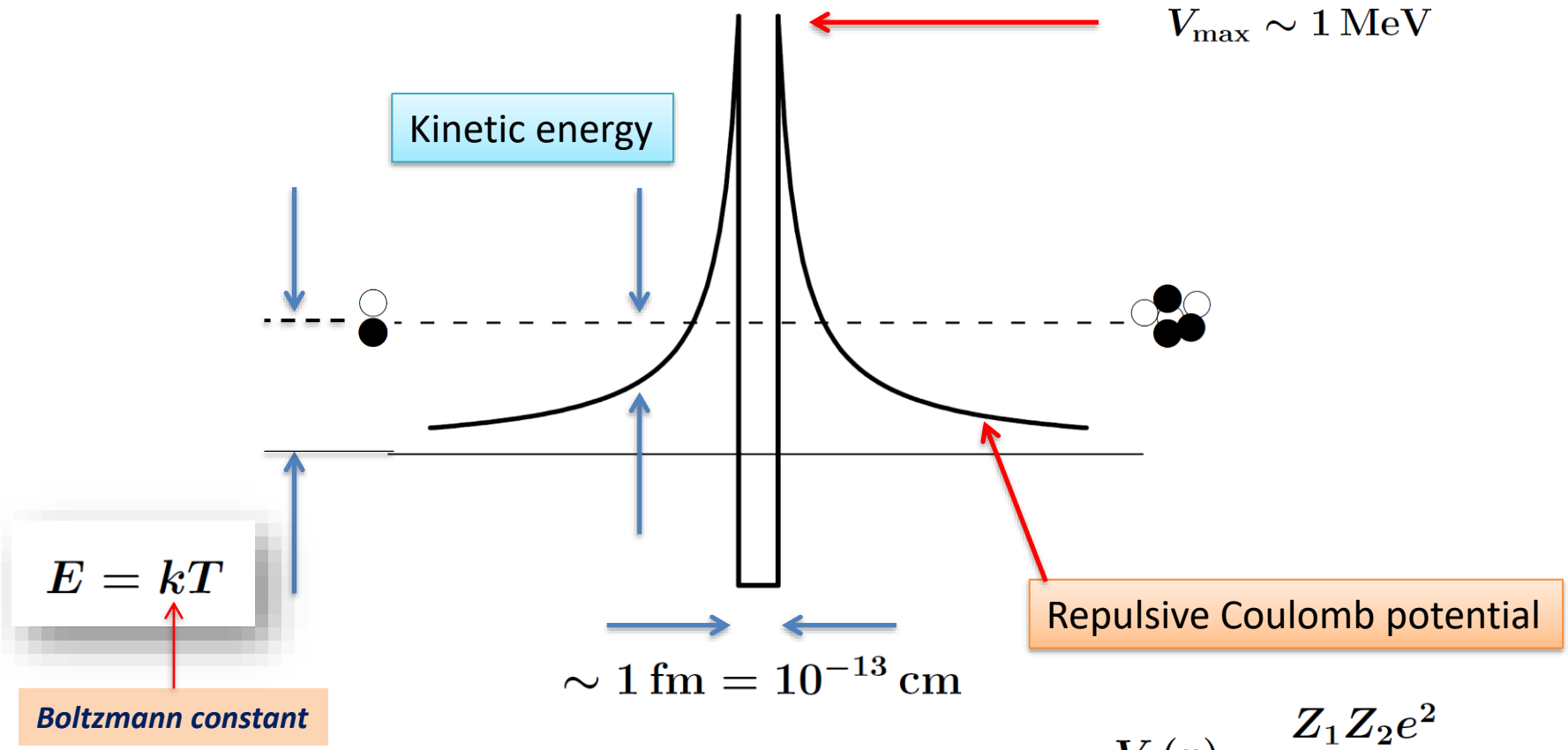


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Nuclear fusion
induced by a neutron flux
in a crystal

$W_n = \frac{k(\Delta x)^2}{2} C_v = \frac{1}{2} \mu \omega^2 R$
 $I = \frac{U}{R}$
 $\langle D \rangle = \frac{n_2 - n_1}{\lambda_1 - \lambda_2}$
 $\vec{a} = 2n_1 + 2n_2$
 $\langle v \rangle = \frac{\Delta S}{\Delta t}$
 $\Delta S = S_2 - S_1$
 $v = \cos \theta \cdot c$
 $A = A_0 e^{-\beta x}$
 $A = n(V_2 - V_1)$
 $A = \frac{m \cdot \Delta T \cdot \mu}{\rho}$
 $Q = \Delta U + A$
 $c = \frac{dQ}{dt}$
 $C = c \cdot \mu$
 $S_2 - S_1 = \int \frac{dQ}{T}$
 $\Psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}$
 $\omega = \sqrt{\omega_0^2 - \beta^2}$
 $v_K = \frac{A}{h}$
 $E = mc^2$
 $h\nu = A + \frac{mv_{max}^2}{2}$
 $\Delta m > 0$
 $\Delta m < 0$
 $m_0 = -$
 $\langle \lambda \rangle = (\sqrt{2\pi} d^2 n)^{-1}$
 $C = c \cdot \mu$
 $R = \alpha \sigma T^4$
 $\alpha = A_0 e^{-\beta t} \cos(\omega t + \alpha)$
 $W = |\Psi|^2$
 $A_m = \frac{b}{T}$
 $b = 2.9 \cdot 10^{-3} \text{ m} \cdot \text{K}$
 $\varphi = \arctan \frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2}$
 $\Delta s = m\lambda_n, m = 0, 1, 2, \dots$
 $A_p = \frac{f_0}{2\beta \sqrt{\omega_0^2 - \beta^2}}$
 $W = \frac{1}{2} m \beta^2 \omega^2$
 $E = A \cos(\omega t - kx)$
 $M = F \cdot d$
 $\Delta \varphi = \frac{2\pi}{\lambda} \Delta x$
 $\rho = nkT$
 $\eta = \frac{1}{3} \rho \langle v \rangle \langle \lambda \rangle$
 $U = \frac{1}{2} \frac{m}{\lambda} RT$
 $\sigma = en(u_n + u_p)$
 $E_n = \frac{h^2}{8ml^2} n^2$
 $\lambda = \frac{h}{p}$
 $\varphi = \frac{W}{Q_0}$
 $\lambda_K = \frac{hc}{A}$
 $W = mgh$
 $F_T = NN$
 $\langle v \rangle = \sqrt{\frac{8kT}{\pi m_0}} = \sqrt{\frac{8RT}{\pi M}}$
 $A = F \Delta s \cos \alpha$
 $\psi(x)$
 $R_1 = \frac{35}{8} \frac{r}{ne}$
 $p = p_0 e$
 $p = \frac{h}{\lambda}$
 $\psi = N\Phi$
 $E_s = -L \frac{dI}{dt}$
 $\langle v \rangle = \sqrt{\frac{8kT}{\pi m_0}} = \sqrt{\frac{8RT}{\pi M}}$
 $A = F \Delta s \cos \alpha$

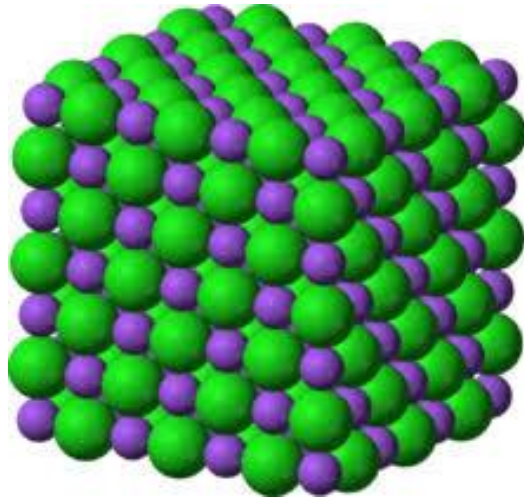


$$V_c(r) = \frac{Z_1 Z_2 e^2}{r}$$

	T (°K)	E	Penetration
room	300	10^{-2} eV	10^{-2600}
sun	15×10^6	1.3 keV	10^{-10}
V_{\max}	10^{10}	1 MeV	1

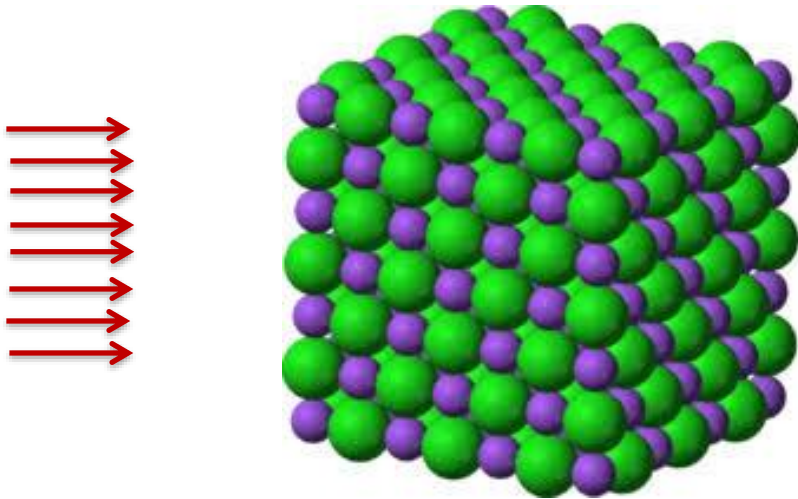
$$P \sim \exp \left(-\frac{2\pi Z_1 Z_2}{137} \sqrt{\frac{mc^2}{2E}} \right)$$

Number of *pp*-pairs in the sun $\sim 10^{57}$



$$N_A = 6.02214076 \times 10^{23} \text{ mol}^{-1}$$

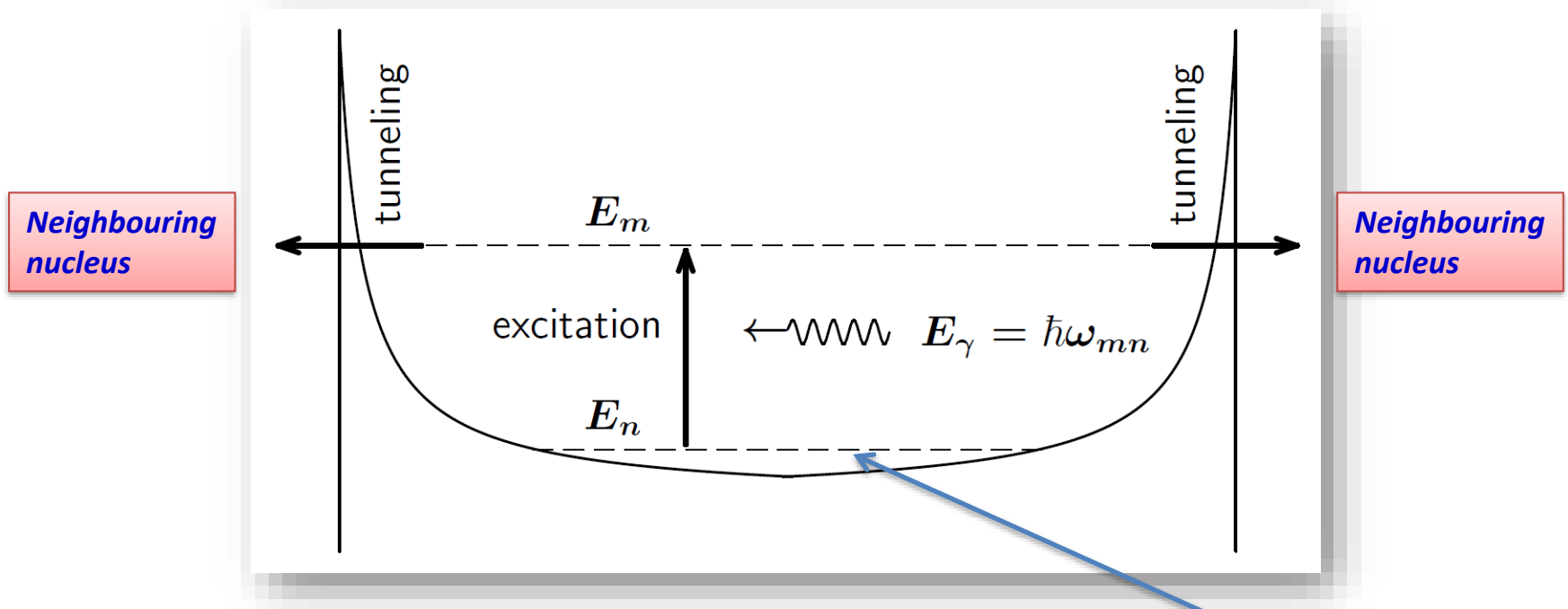
whole crystal	single pair
1 fusion/hour	$\frac{1}{N_A \cdot 3600 \text{ s}} \approx 10^{-27} \text{ s}^{-1}$



$$N_A = 6.02214076 \times 10^{23} \text{ mol}^{-1}$$

whole crystal	single pair
1 fusion/hour	$\frac{1}{N_A \cdot 3600 \text{ s}} \approx 10^{-27} \text{ s}^{-1}$

Electromagnetically induced fusion (in a crystal)



V. B. Belyaev, M. B. Miller, J. Otto, and S. A. Rakityansky
Nuclear fusion induced by x rays in a crystal
Phys. Rev. C 93, 034622 (2016)

Exposing the solid compound LiD (lithium deuteride) to X rays for the duration of 111 h, **we detect 88 events of nuclear fusion** $d + {}^6\text{Li} \rightarrow {}^8\text{Be}^*$

Nuclear fusion induced by x rays in a crystal

V. B. Belyaev,^{1,*} M. B. Miller,² J. Otto,³ and S. A. Rakityansky^{3,†}

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²*Institute for Physical and Technical Problems, Dubna 141980, Russia*

³*Department of Physics, University of Pretoria, Pretoria, Hatfield 0028, South Africa*

(Received 28 October 2015; published 28 March 2016)

The nuclei that constitute a crystalline lattice oscillate relative to each other with a very low energy that is not sufficient to penetrate through the Coulomb barriers separating them. An additional energy, which is needed to tunnel through the barrier and fuse, can be supplied by external electromagnetic waves (x rays or synchrotron radiation). Exposing the solid compound LiD (lithium deuteride) to x rays for the duration of 111 h, we detect 88 events of nuclear fusion $d + {}^6\text{Li} \rightarrow {}^8\text{Be}^*$. Our theoretical estimate agrees with what we observed. One possible application of the phenomenon we found is in measurements of the rates of various nuclear reactions (not necessarily fusion) at extremely low energies inaccessible in accelerator experiments.

DOI: [10.1103/PhysRevC.93.034622](https://doi.org/10.1103/PhysRevC.93.034622)

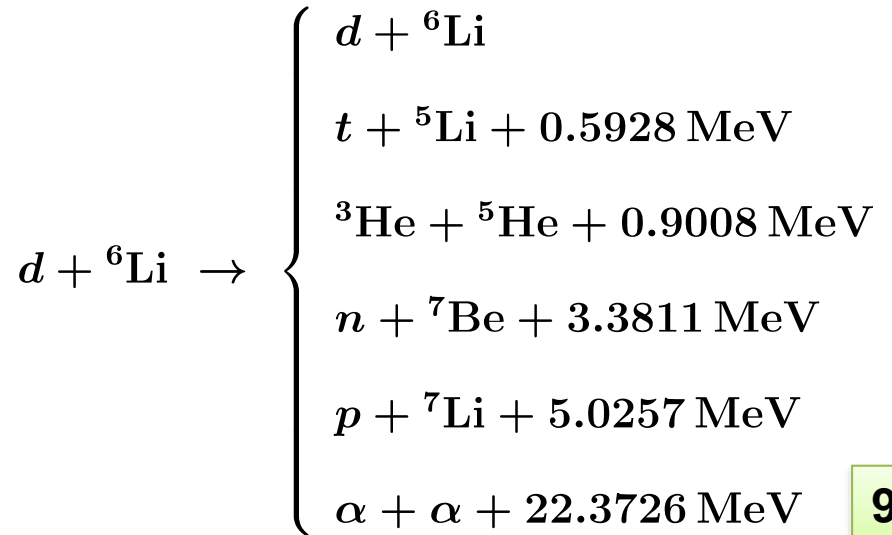
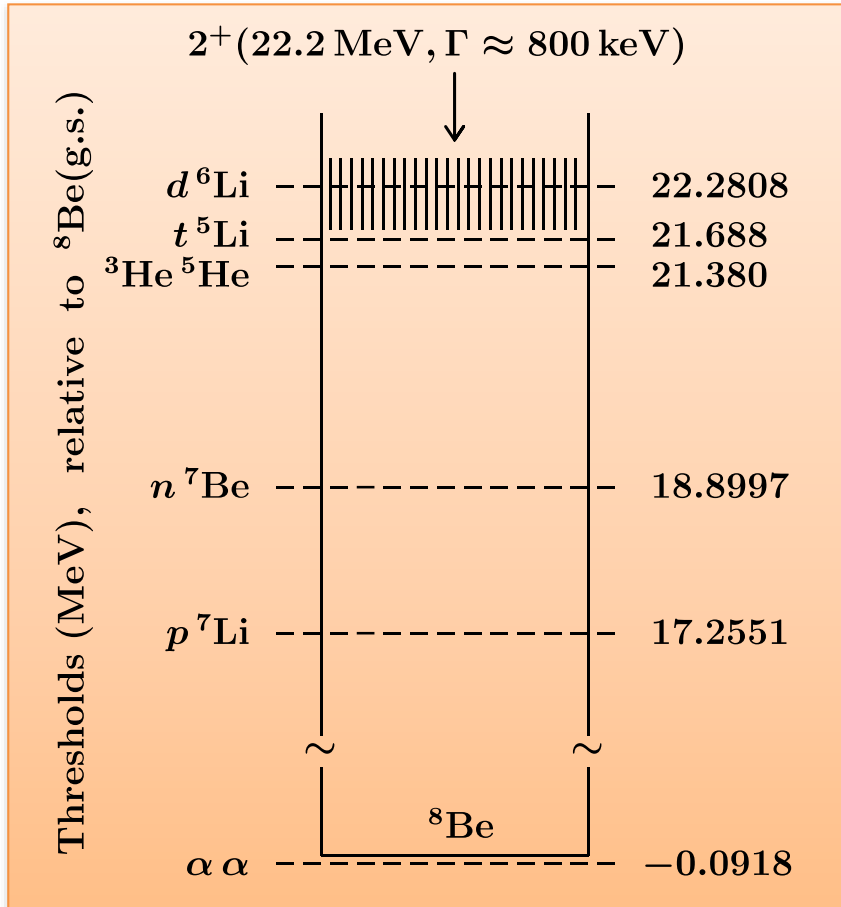
I. INTRODUCTION

Fusion of two atomic nuclei is possible if they approach each other at a short distance ($\sim 10^{-13}$ cm). To come that close, they need to go through a Coulomb barrier of the height of a few MeV. The penetration probability for such a barrier at room temperature (energy of relative motion, ~ 10 meV) is practically 0 ($\sim 10^{-2600}$ [1]), but this probability rapidly increases when the kinetic energy of the nuclei increases. For example, for the dd system at an energy of 30 keV the penetration probability becomes $\sim 10^{-3}$ [1]. Therefore, the obvious way to fuse the nuclei is to raise the temperature of their mixture. In this way so-called thermonuclear reactions occur in stellar bodies, in nuclear weapons, and in the TOKAMAK [2].

such a probability, one muon can help with fusion, i.e., can catalyze the fusion of many nuclear pairs before it decays (muon lifetime, $\sim 2 \times 10^{-6}$ s). Muon-catalyzed fusion has been observed and well studied both experimentally and theoretically (see, for example, Refs. [6] and [7]) but turned out to be inefficient as a new source for energy production.

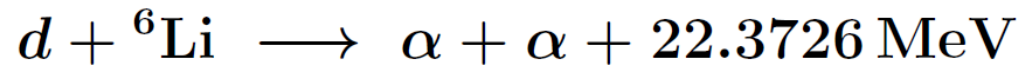
In the present paper, we suggest and experimentally explore yet another possible approach to fusion of light nuclei. The idea is to make a crystal out of the atoms whose nuclei we want to fuse. In this crystal, the nuclei sit next to each other at an atomic distance and oscillate around the equilibrium positions. A crystal is just a huge molecule, and of course the probability of spontaneous fusion of neighboring nuclei is negligible, the same as in ordinary molecules. An experiment aimed at observing spontaneous fusion in the lithium deuteride

$d\ ^6\text{Li}$ -- system



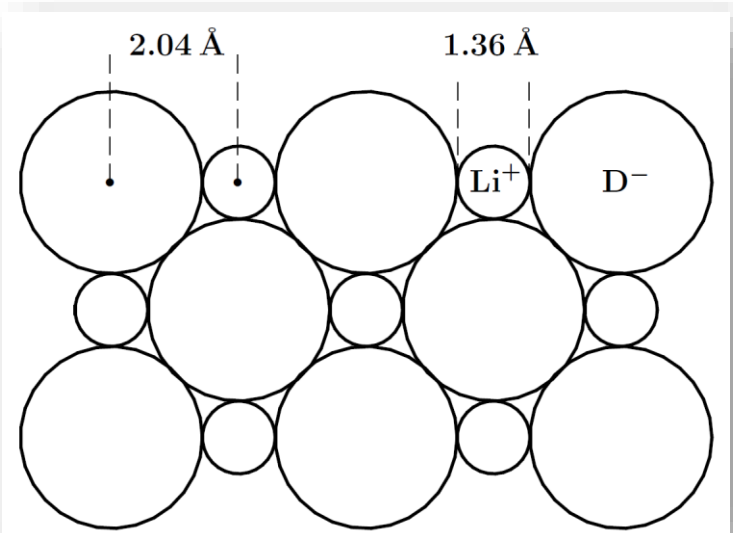
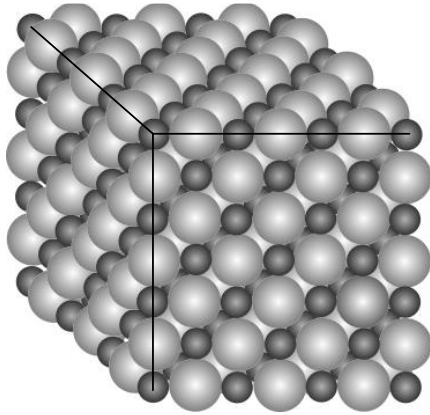
96%

Barrier penetration



LiD - crystal

${}^6\text{Li} - {}^2\text{H}$

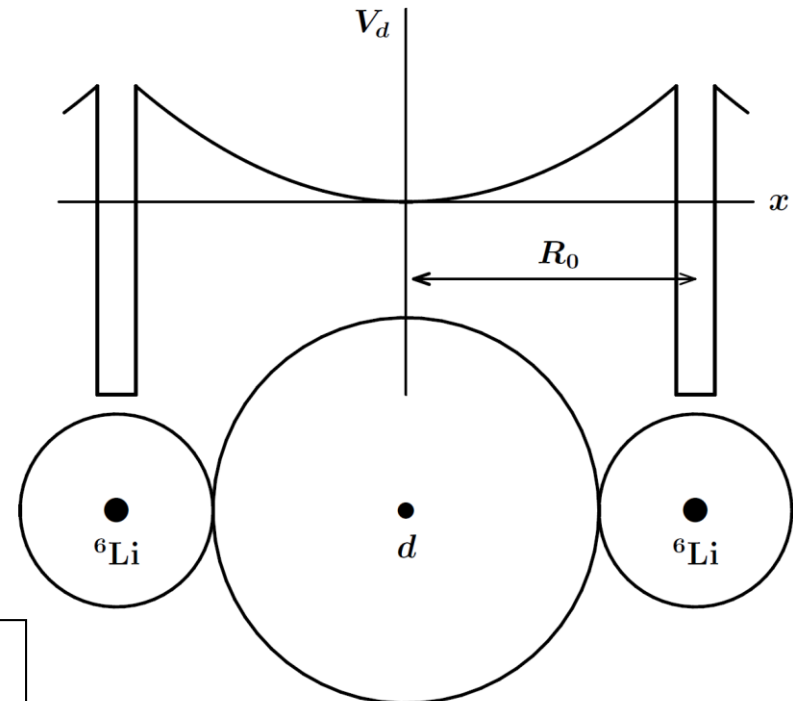


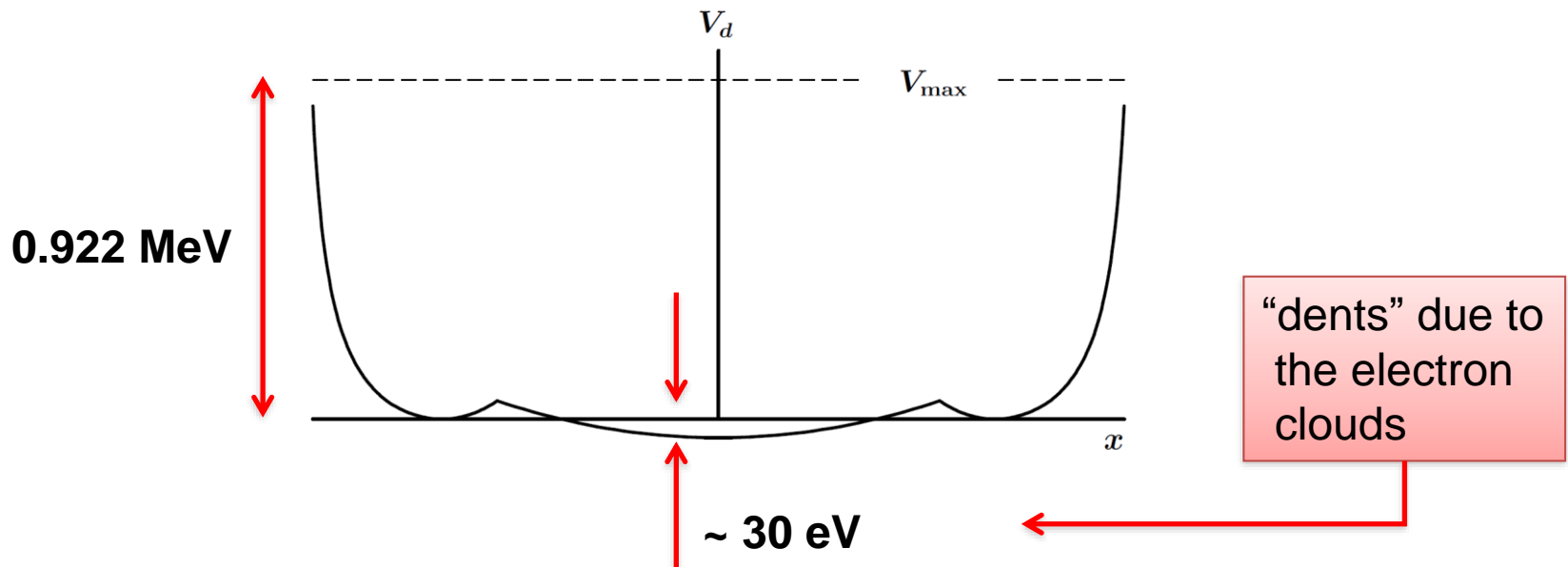
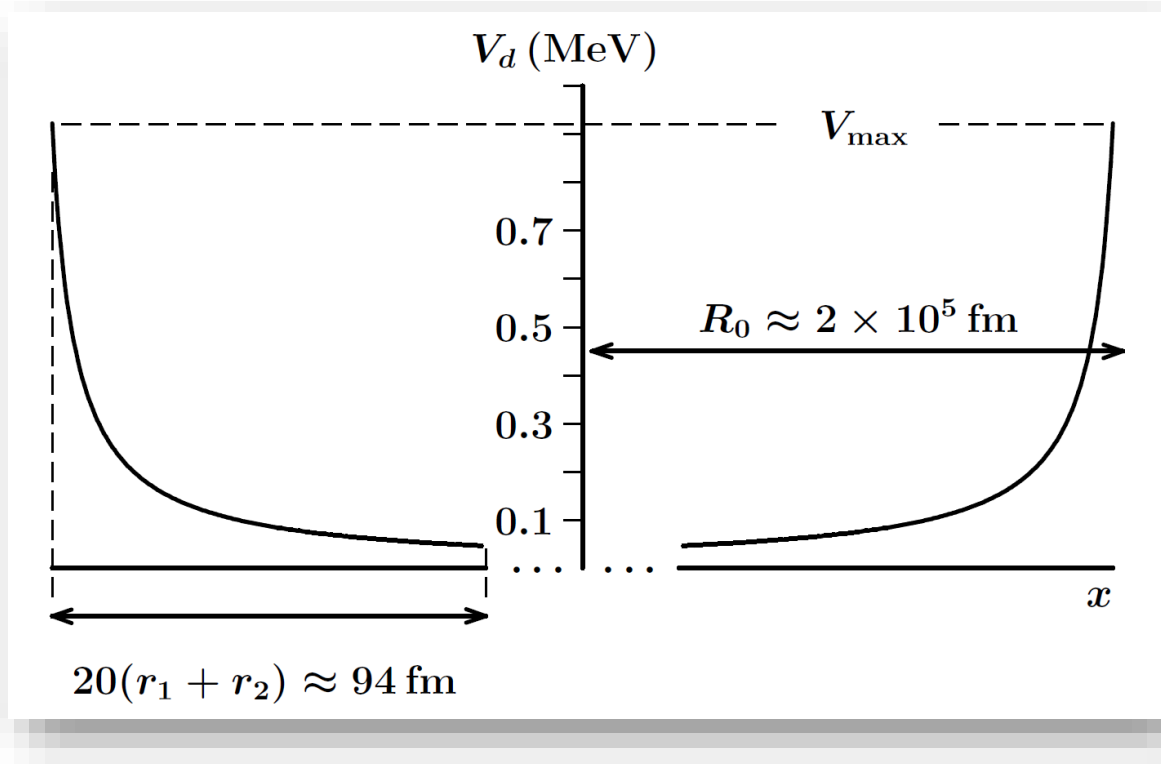
$$V_d(x) = \frac{Z_1 Z_2 e^2}{R_0 - x} + \frac{Z_1 Z_2 e^2}{R_0 + x}$$

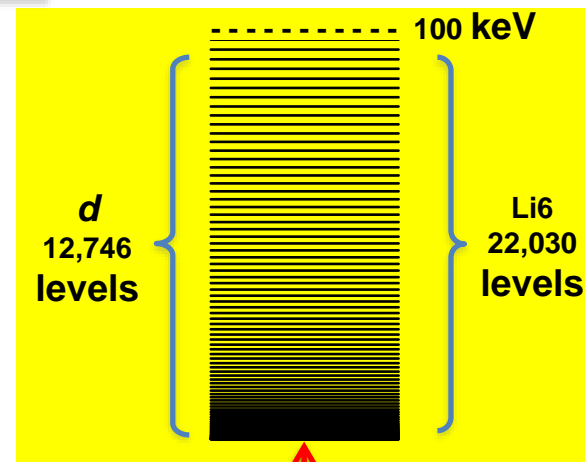
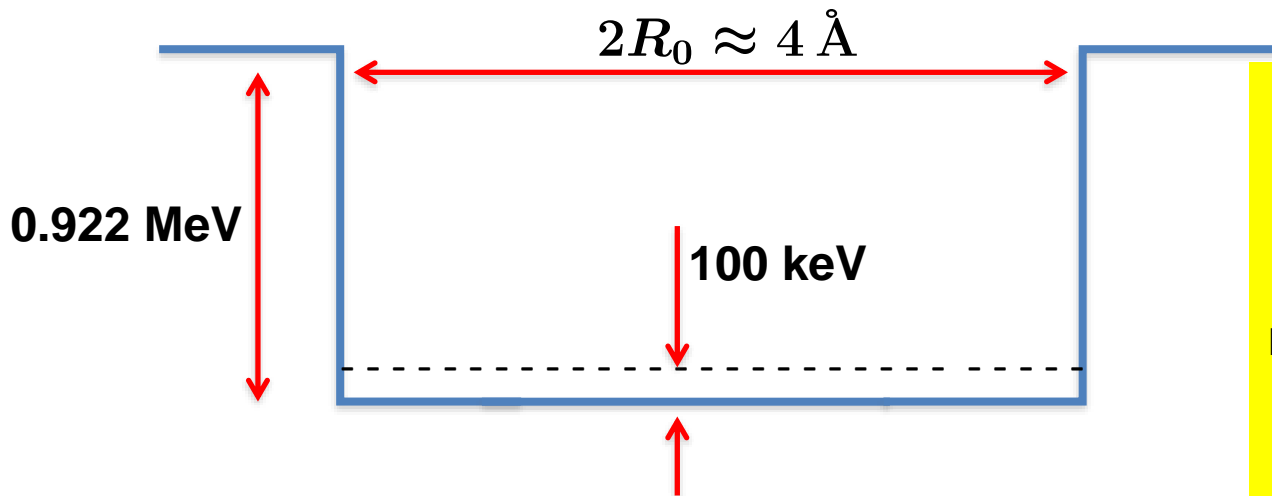
$$V_{\max} \approx \frac{Z_1 Z_2 e^2}{r_1 + r_2} \approx 0.922 \text{ MeV}$$

$$V_{\min} = \frac{2Z_1 Z_2 e^2}{R_0} \approx 42.35 \text{ eV}$$

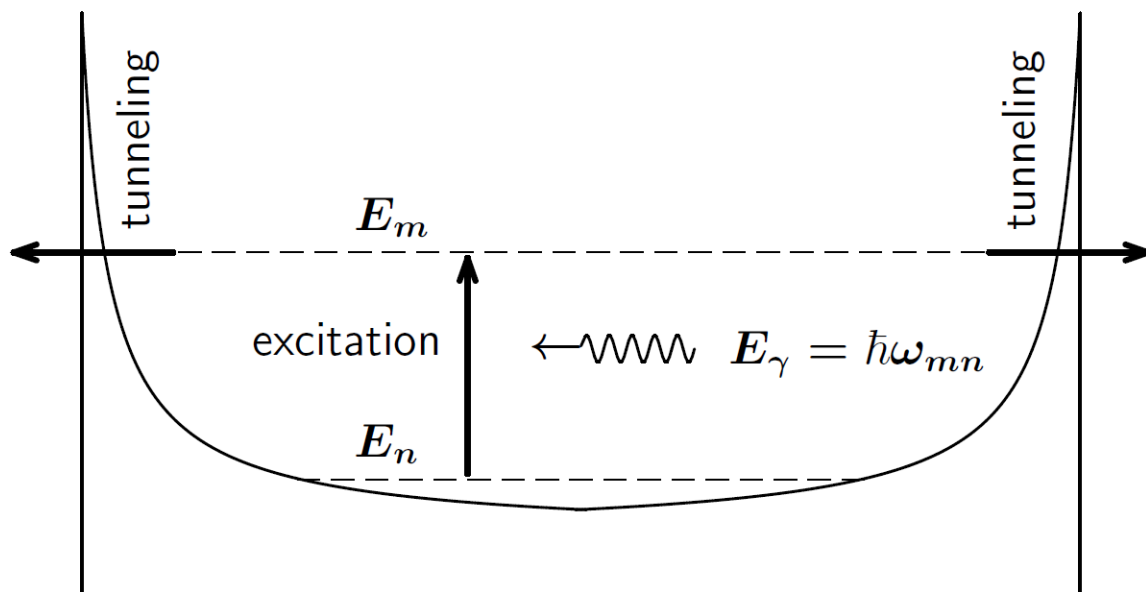
$$r_1 = 2.1424 \text{ fm} , \quad r_2 = 2.5432 \text{ fm}$$





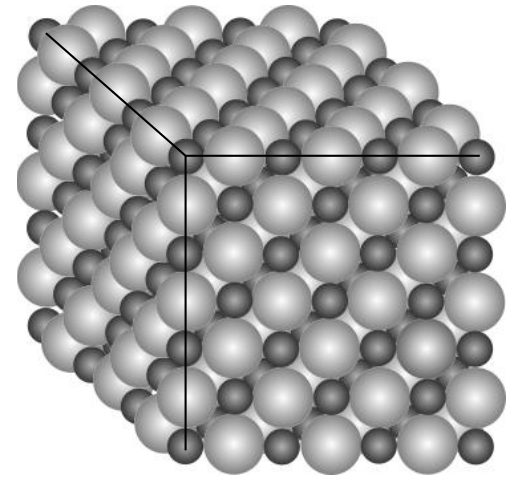
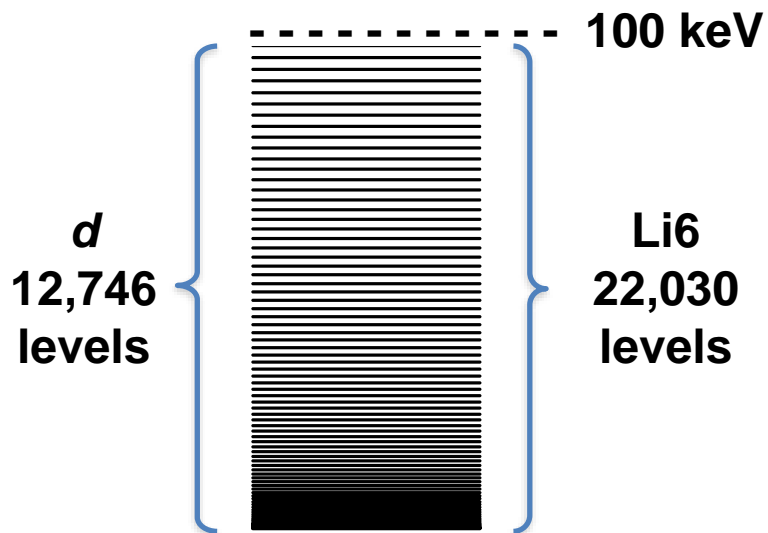


$$E_n = \frac{\pi^2 \hbar^2 n^2}{8\mu_d R_0^2}, \quad \psi_n(x) = \frac{1}{\sqrt{R_0}} \sin \frac{\pi n (R_0 + x)}{2R_0}, \quad n = 1, 2, 3, \dots$$

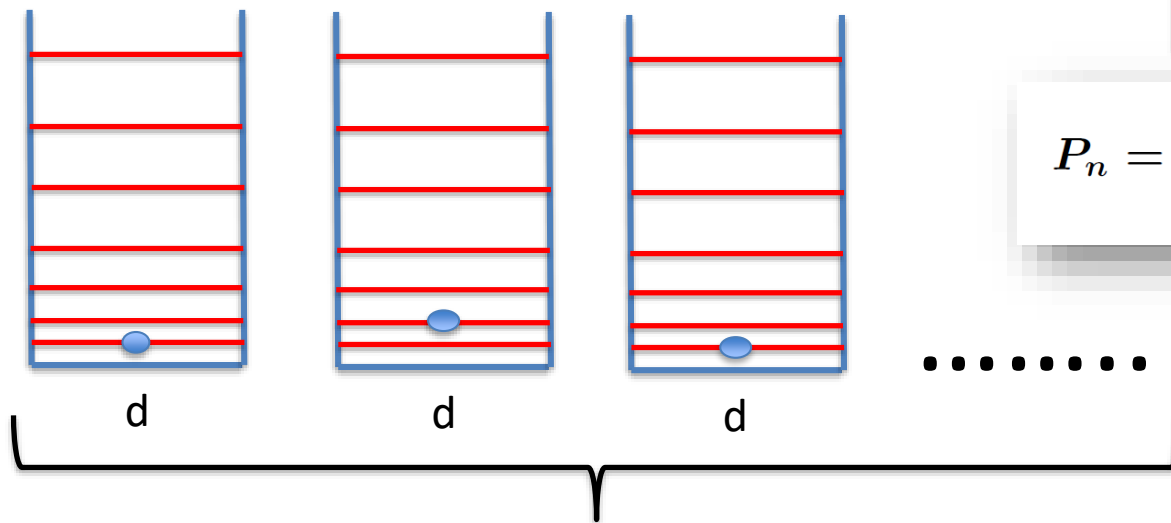


$$E_1(d) \approx 0.6 \text{ meV}$$

$$E_1(\text{Li}) \approx 0.2 \text{ meV}$$



LiD - crystal



Avogadro number of nuclei $\sim 10^{23}$

$$P_n = \frac{\exp(-E_n/k_B T)}{\sum_{j=1}^{\infty} \exp(-E_j/k_B T)}$$

Boltzmann distribution

$T = 300 \text{ }^\circ\text{K}$

$$\langle E \rangle_d = \sum_{n=1}^{\infty} E_n P_n \approx 14.2 \text{ meV}$$

$$\langle E \rangle_{\text{Li}} \approx 13.6 \text{ meV}$$

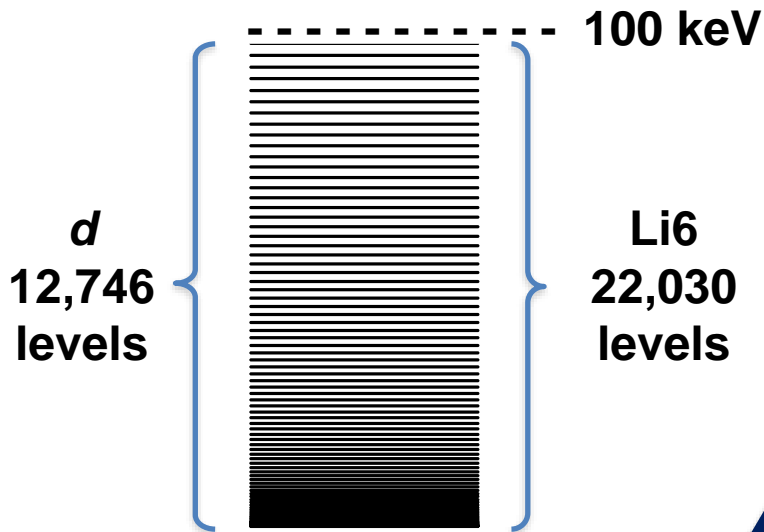
$$T = 300 \text{ }^\circ\text{K} : \quad \langle E \rangle_d = \sum_{n=1}^{\infty} E_n P_n \approx 14.2 \text{ meV}$$

$$\langle E \rangle_{\text{Li}} \approx 13.6 \text{ meV}$$

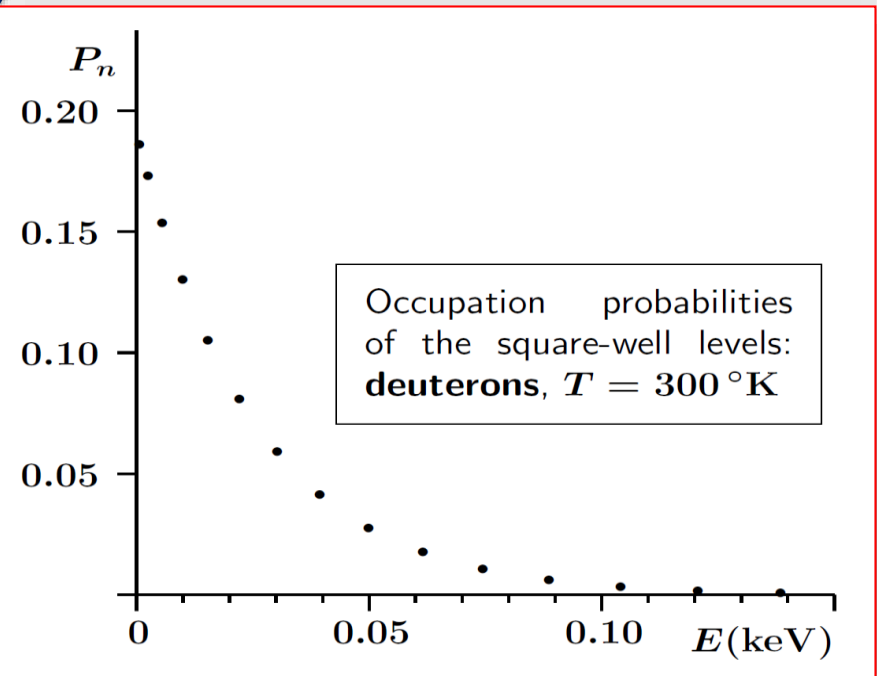
Population of the energy levels

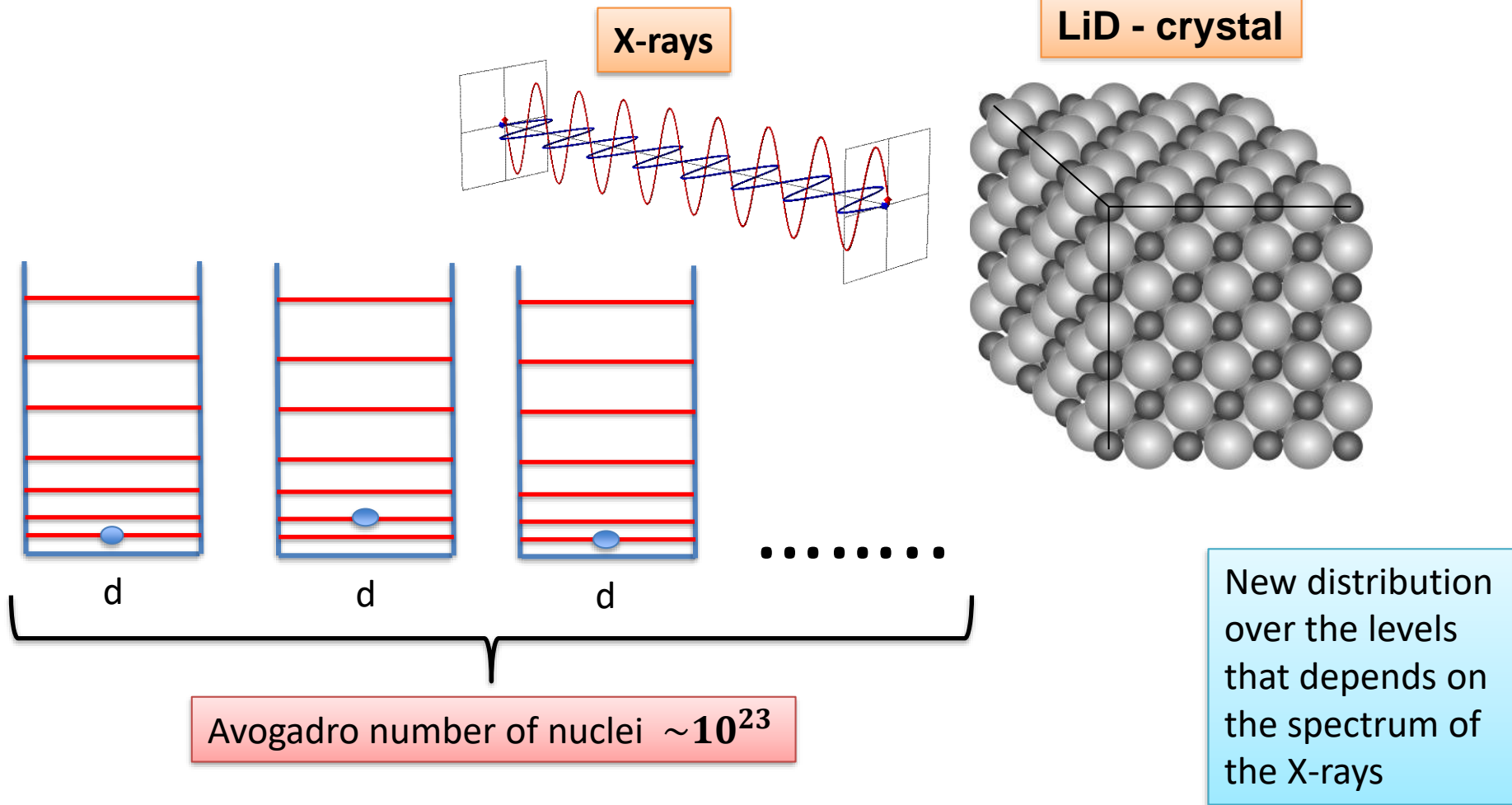
Boltzmann distribution

$$P_n = \frac{\exp(-E_n/k_B T)}{\sum_{j=1}^{\infty} \exp(-E_j/k_B T)}$$



$$E_n = 100 \text{ keV} \longrightarrow P_n \sim 10^{-1700000}$$





Under the X-ray radiation the distribution is changing

master equation

$$\frac{dP_m}{dt} = \sum_{n \neq m} p_{mn} P_n - \sum_{n \neq m} p_{nm} P_m + \sum_{n > m} p_{mn}^{sp} P_n - \sum_{n < m} p_{nm}^{sp} P_m$$

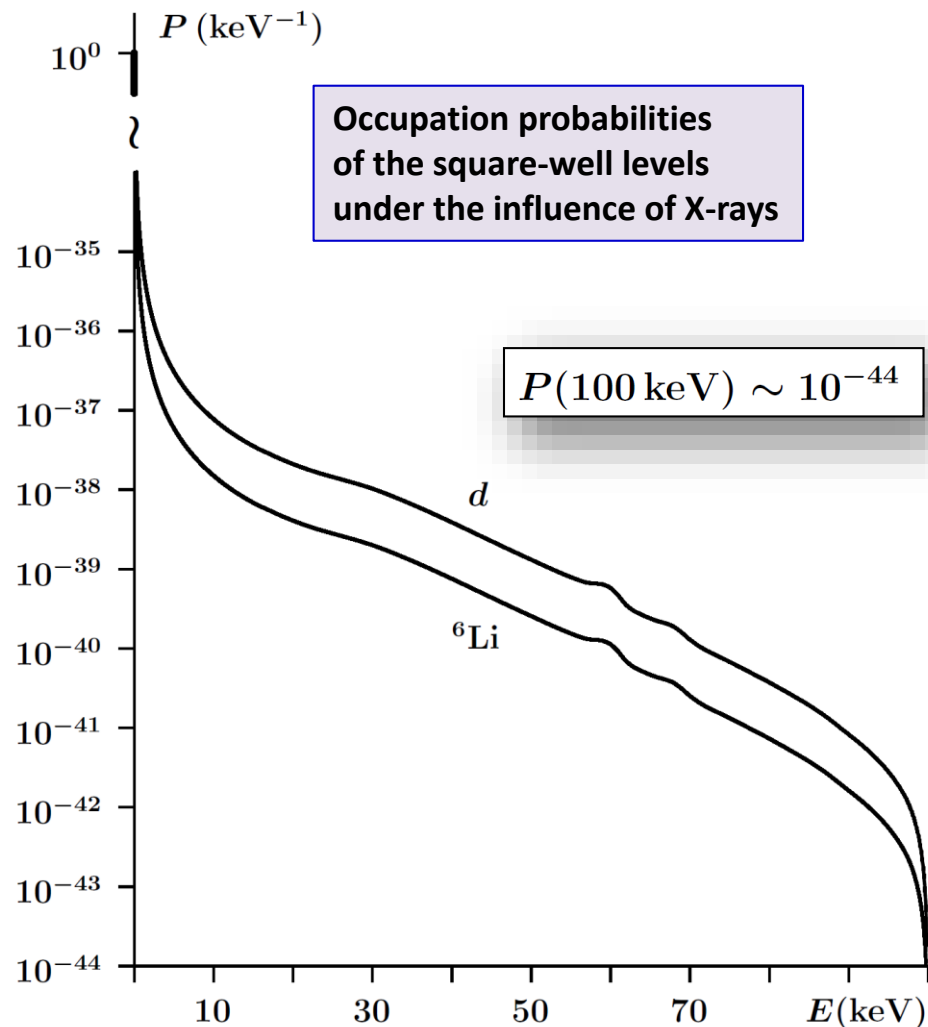
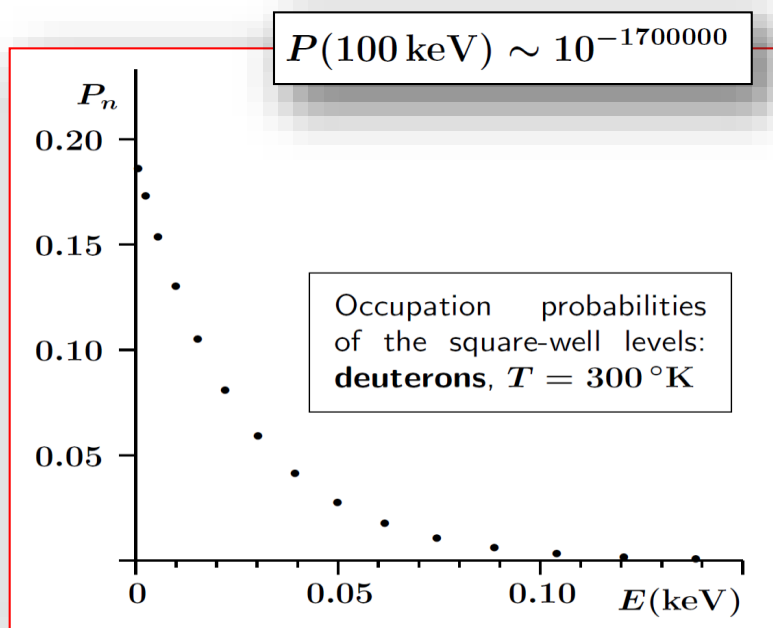
We only need the stationary distribution:

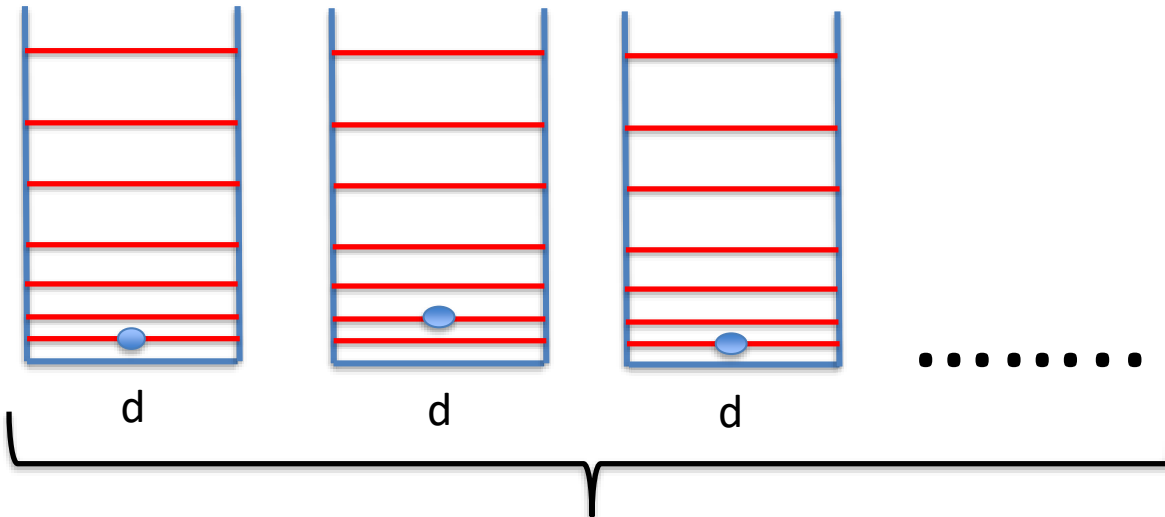
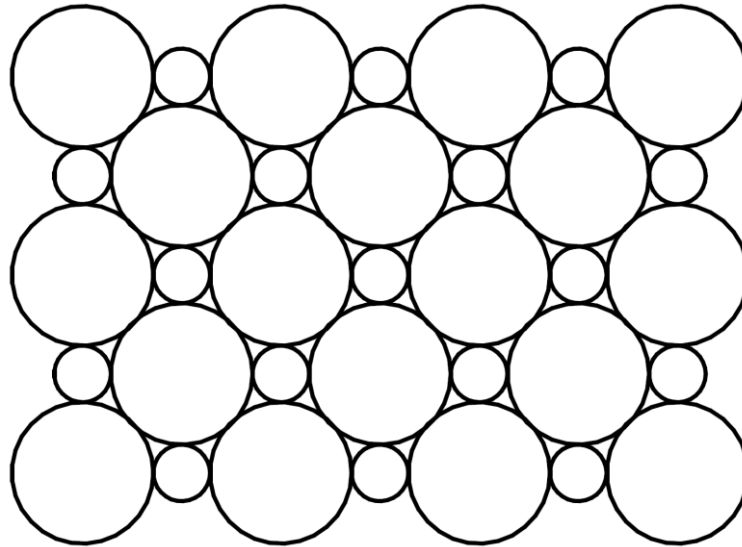
$$0 = \sum_{n \neq m} p_{mn} P_n - \sum_{n \neq m} p_{nm} P_m + \sum_{n > m} p_{mn}^{\text{sp}} P_n - \sum_{n < m} p_{nm}^{\text{sp}} P_m$$

normalization
condition

$$\sum_n P_n = 1$$

This linear system can be solved iteratively





Avogadro number of nuclei $\sim 10^{23}$

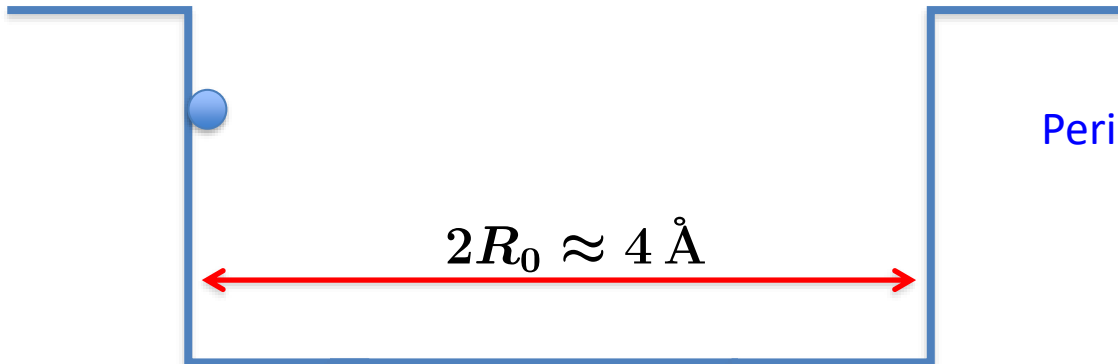
New distribution over the levels that depends on the spectrum of the neutron flux

penetration probability

$$T(E) = \frac{2\pi\eta}{\exp(2\pi\eta) - 1} \xrightarrow{E \rightarrow 0} 2\pi\eta \exp(-2\pi\eta)$$

Sommerfeld parameter:

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar} \sqrt{\frac{\mu}{2E}}$$



Period of collisions $t = \frac{4R_0}{v}$

frequency $\nu = \frac{v}{4R_0}$

Fusion rate

$$W_d(E) = T\nu = \frac{T(E)}{4R_0} \sqrt{\frac{2E}{\mu_d}}$$

Similarly, the rate

$$W_{\text{Li}}(E)$$

can be found

Average rates
for the ensemble

$$\langle W_d \rangle = \sum_n W_d(E_n^{(d)}) P_d(E_n^{(d)})$$

$$\langle W_{\text{Li}} \rangle = \sum_n W_{\text{Li}}(E_n^{(\text{Li})}) P_{\text{Li}}(E_n^{(\text{Li})})$$

$$\langle W_d \rangle \approx 2.4 \times 10^{-26} \text{ s}^{-1}, \quad \langle W_{\text{Li}} \rangle \approx 4.6 \times 10^{-27} \text{ s}^{-1}$$

“Bulk” rate
for the sample



$$R = 6 \frac{M}{m} N_A f_2 f_6 (\langle W_d \rangle + \langle W_{\text{Li}} \rangle)$$

molar mass

$$m = f_1 1 \text{ g} + f_2 2 \text{ g} + f_6 6 \text{ g} + f_7 7 \text{ g}$$

$$f_2 = 0.98$$

$$f_6 = 0.0759$$

$$M = 0.61 \text{ g}$$

$$R \approx 5.2 \times 10^{-4} \text{ s}^{-1}$$



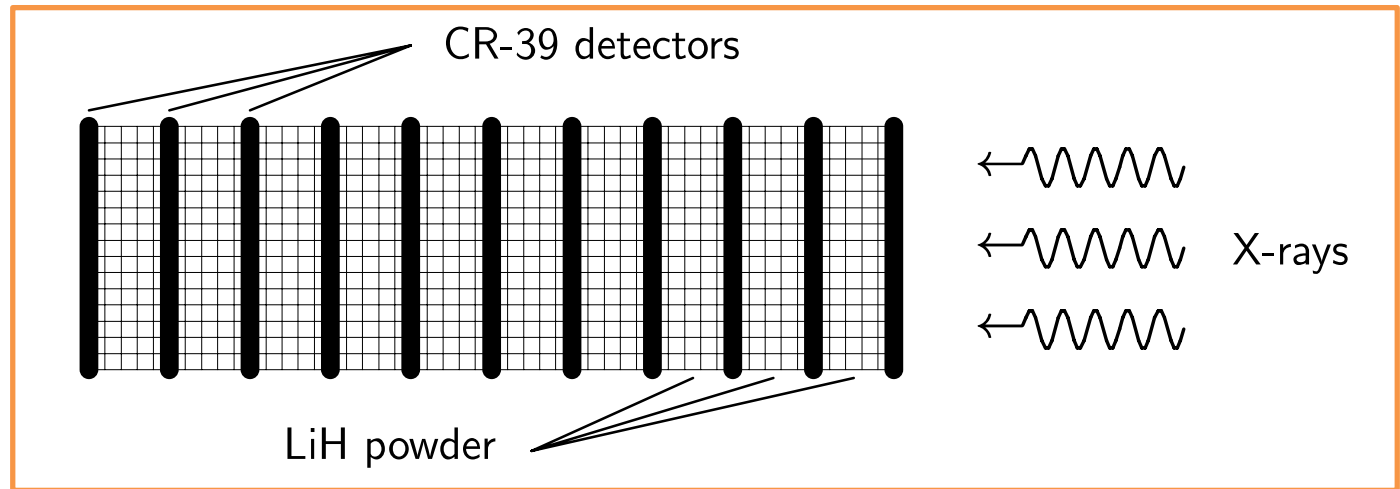
theory

Exposure time $t = 111$ hours
Expected events: $N = 207$

Experiment: $N > 88$

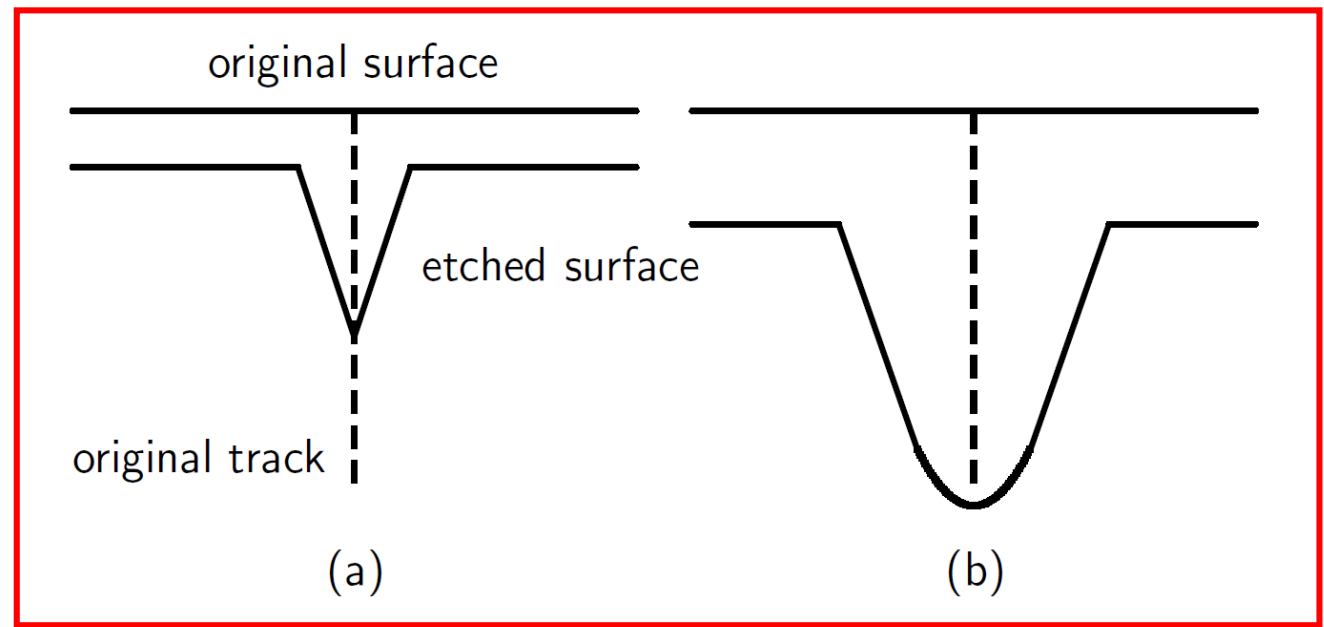
Experiment

85 detector
plates
1cm X 1cm



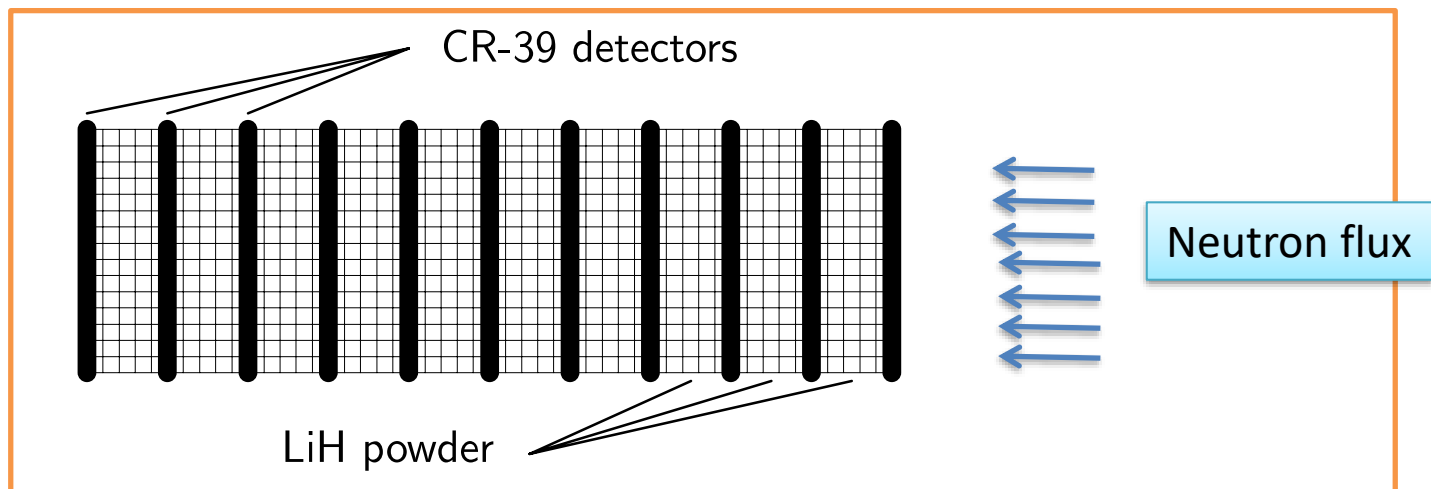
Etching:
6.25 mol/L
NaOH
70 ° C
8 hours

$E > 6 \text{ MeV}$



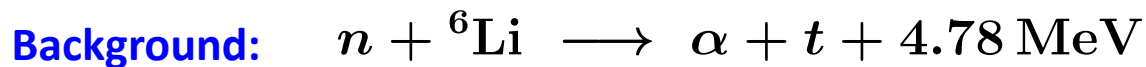
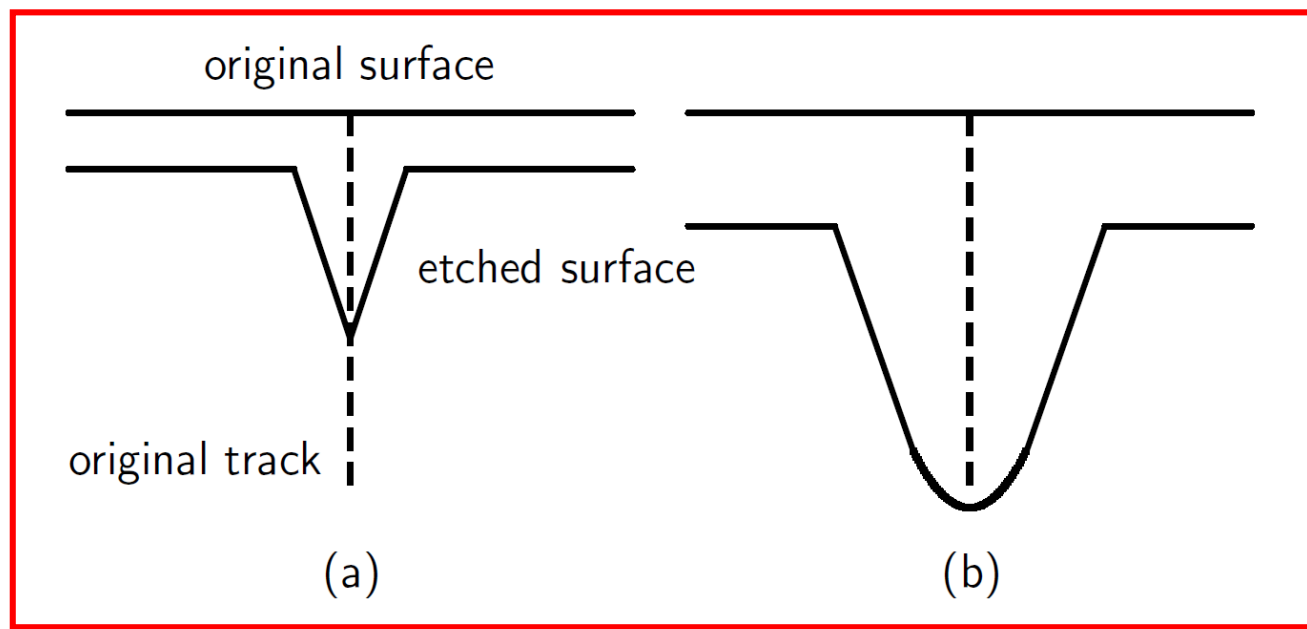
Background: $n + {}^6\text{Li} \longrightarrow \alpha + t + 4.78 \text{ MeV}$

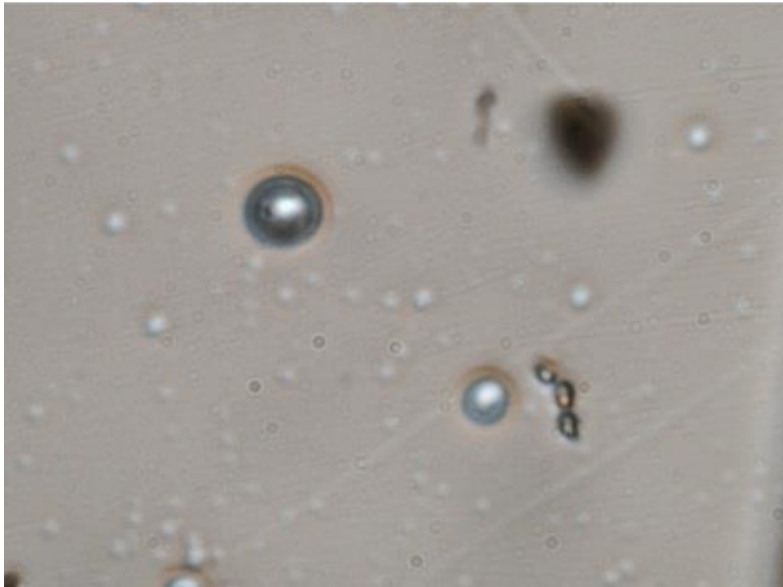
Experiment



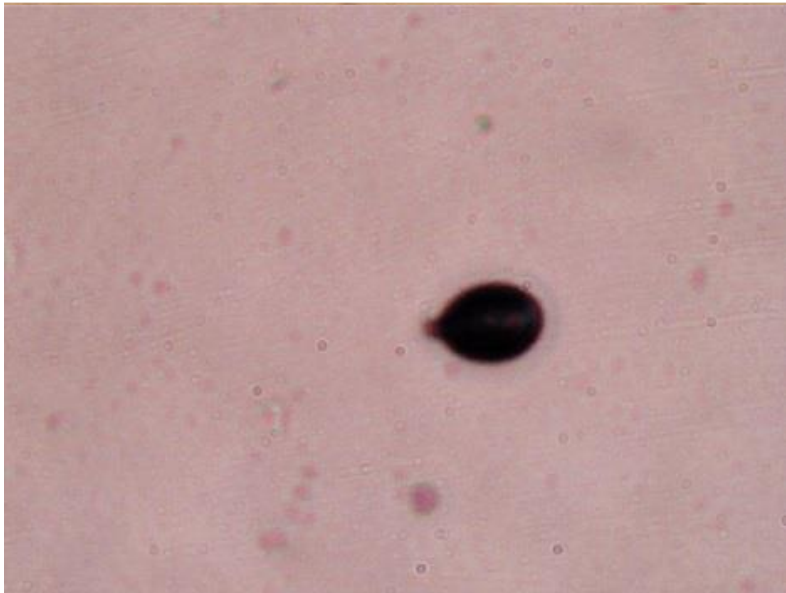
Etching:
6.25 mol/L
NaOH
70 ° C
8 hours

$E > 6$ MeV

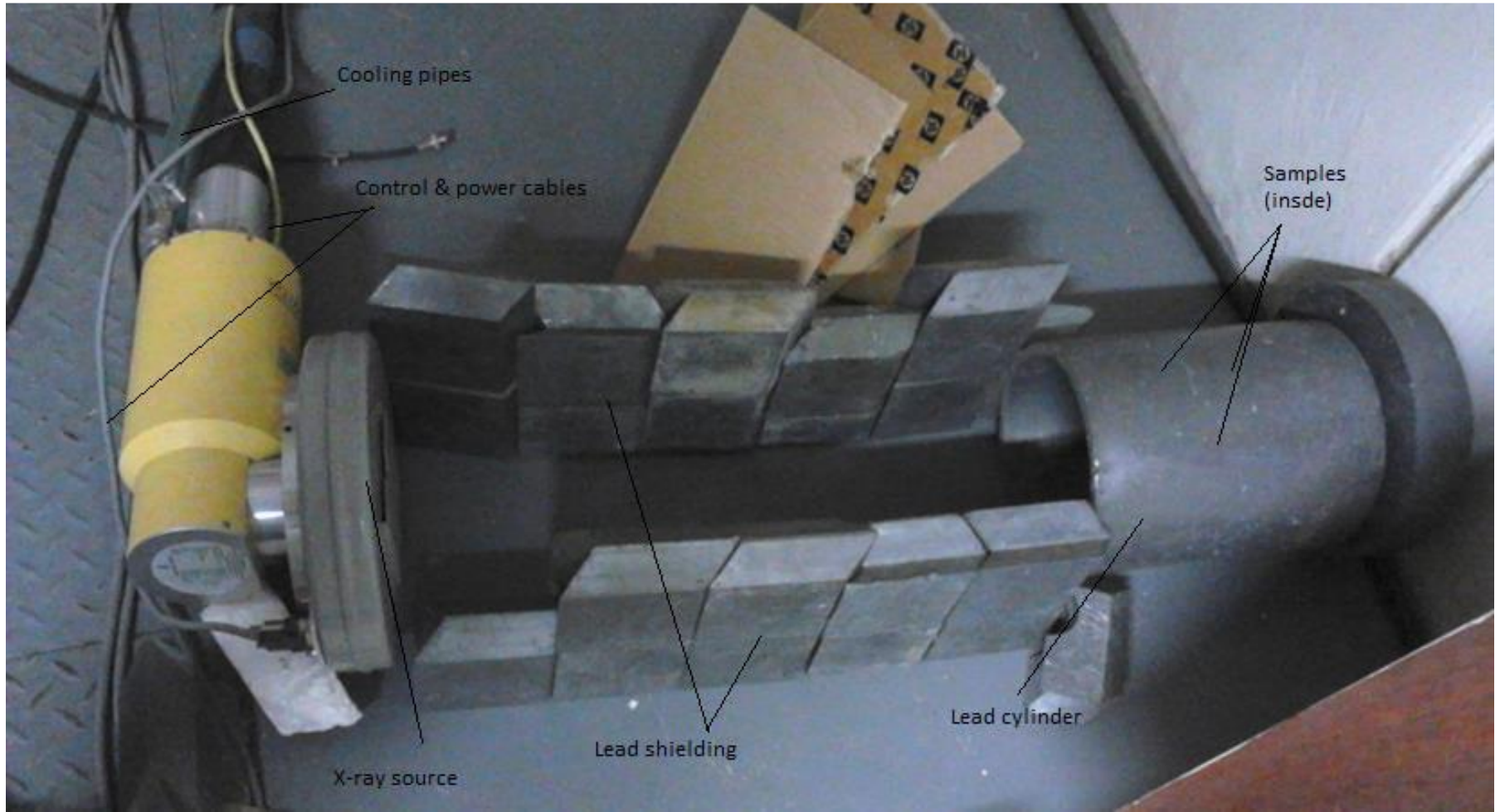




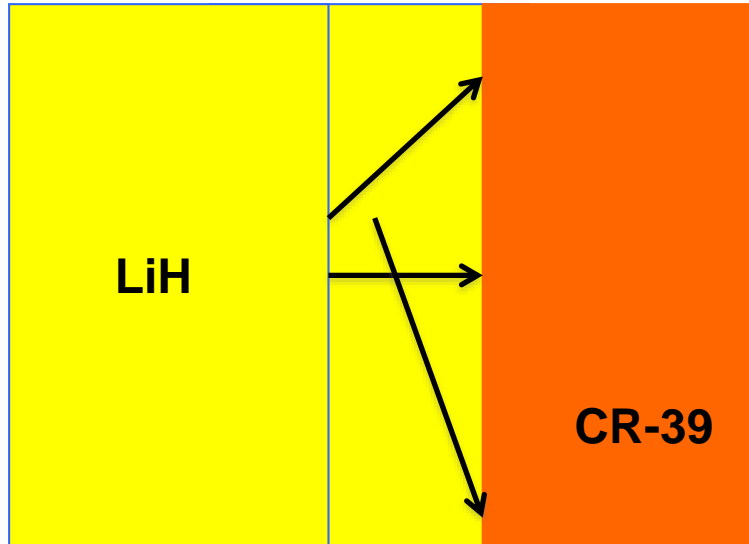
Low-energy
tracks



$E > 6 \text{ MeV}$



Efficiency of detecting



We cannot register all the fusion events because the α -particles slow down when passing through the material



$$M = 0.61 \text{ g}$$

Theory: $N = 207$

Experiment: $N > 88$

$$88 \approx 40\% \rightarrow N \sim 220$$

SUMMARY

- **Neutrons can excite the oscillations of the nuclei relative to each other in a crystal**
- **Low fusion probabilities are magnified by the macroscopic number of the pairs**
- **Such a phenomenon could be used to study the low-energy nuclear reactions**



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